A THEORETICAL REVIEW OF SPECIALISED CONTENT KNOWLEDGE

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This work is a bibliographical review of Specialised Content Knowledge from the model of Mathematical Knowledge for Teaching. It offers a discussion of the most frequent definitions for this subdomain. We work with two examples of specific tasks which, according to the authors, require specialised knowledge on the part of the teacher. We identify the essential characteristics of the mathematical knowledge involved in these tasks and contrast these with the features commonly employed to identify specialised knowledge. We conclude with a discussion of the nature of specialised knowledge, which serves as the starting point for several papers by our research group to be presented to this working group.

Keywords: Specialised Content Knowledge, Mathematical Knowledge for Teaching.

INTRODUCTION

What does a teacher need to know to teach mathematics? What mathematical knowledge does the teacher require to teach a specific topic? How and where can teachers use this knowledge in practice? Questions like these have prompted numerous research projects aimed at studying the *ideal* knowledge and skills which a mathematics teacher should possess. In particular, a research group based at the University of Michigan has spent several years working on a scheme which allows them to categorise the typology of mathematical knowledge observed in, and required by, education: Mathematical Knowledge for Teaching (MKT).

MKT refines the map originally put forward by Shulman (1986) into subdomains, thus: the superordinate 'Subject Matter Knowledge' domain is subdivided into 'Common Content Knowledge' (CCK), 'Specialized Content Knowledge' (SCK), and 'Horizon Content Knowledge' (HCK); 'Pedagogical Content Knowledge' is in its turn subdivided into 'Knowledge of Content and Students', (KCS), ''Knowledge of Content and Teaching' (KCT) and 'Knowledge of Content and Curriculum' (KCC). This breaking down of domains into more finely defined sub-categories owes as much to lesson

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observation as to reflection on what sort of knowledge teachers should have, and the demands, in terms of mathematical reasoning, intuition, understanding and skill, the profession places upon them (Ball, Thames, & Phelps, 2008).

The aim of this short research is to take a closer look at SCK, and map out the advances made in the field, its nature and the difficulties that arise when it is applied to systematising teachers' mathematical knowledge.

DEFINITION OF SCK

One of the main contributions of MKT, according to its authors, is the identification of knowledge in terms that are purely mathematical and specific to the profession, SCK. This has been largely well received by the research community in that it specifies the teacher's knowledge. However, there are also some drawbacks which make it difficult to observe and analyse.

In this section we present the results of a wide-ranging literature review on what, from the point of view of the definition, is understood by SCK, drawing on the work of various authors who have used MKT in their research, whether seeking a better understanding of this subdomain or aiming to develop it in some way. Our intention is to give an overview of a collection of studies, looking specifically at how the definition has been adapted and developed over time.

Our starting point is the work of Ball *et al* (2008) in which the definition of SCK is predicated on notions of the profession and Common Content Knowledge, a practice followed by many subsequent authors (Hill *et al*, 2008; Delaney, Ball, Hill, Schilling, & Zopf, 2008; Hill, Ball, & Schilling, 2008; Krauss, Baumert, & Blum, 2008; Knapp, Bomer, & Moore, 2008; Carreño, & Climent, 2009; Suzuka *et al*, 2009; Kazemi *et al*, 2009; Markworth, Goodwin, & Glisson, 2009; Rivas, Godino, & Konic, 2009; Godino, 2009; Van, 2009; Godino, Gonzato, & Fernández, 2010; Sosa, & Carrillo, 2010; Ribeiro, Monteiro, & Carrillo, 2010; Castro, Godino, & Rivas, 2011; Herbst, & Kosko, 2012; Rivas, Godino, & Castro, 2012). However, none of these definitions specifies the nature of the knowledge in itself, but rather they all evoke external agencies.

Definitions alluding to professional demands, tend to make reference to the mathematical knowledge and skills unique to education, and which are generally not used in other contexts. Education requires knowledge beyond the pupils' mode of thinking. This implies a particular way of unpacking mathematical knowledge which is not necessary (or even desirable) in other professions (Ball *et al*, 2008).

Great emphasis is placed on the insistence that this kind of knowledge pertains exclusively to the ambit of mathematics teaching, and is not required in other professions. Nevertheless, one might justly ask how it is that we know that a certain kind of knowledge is not required in other professions. Is it necessary perhaps to check what kind of mathematical knowledge is used in each profession?

Fortunately, an indication of how this task might be undertaken is offered by such definitions themselves. The use of the term ‘skills’ indicates what, until this point, has been lacking in determining the nature of the knowledge involved in SCK. In other words, the definitions of SCK tend to be phrased in terms of what having this knowledge enables one to do: “responding to students’ ‘why’ questions, […] choosing and developing useable definitions, modifying tasks to be either easier or harder” (Ball *et al*, 2008, p. 400), to mention just a few.

Drawing on the work of Rivas *et al* (2012), amongst the skills which can be attributed to this kind of knowledge are selecting and designing class activities, and making representations and giving explanations of curricular items. Suzuka *et al* (2009) emphasise that one skill demanded by SCK is that of interpreting mathematical productions, both those generated by students and those to be found in materials.

From the above, then, it follows that SCK is defined as unique to teachers in that the tasks it allocates to them are indeed specific to mathematics teachers. Nevertheless, it seems to us that there remains the question of whether the mathematical knowledge which allows these tasks to be successfully performed is shared by other professions.

The other tendency which is frequently deployed when defining SCK is comparison with CCK. CCK is defined as the knowledge required in order to solve such tasks as are given to pupils. Other definitions describe it as the knowledge held by a well-educated adult at the educational level in question. Markworth *et al* (2009) symbolically define SCK as “content knowledge needed for the teaching of mathematics, *beyond* the common content knowledge needed by others” (pp. 69). Hence it is knowledge that the pupil may not necessarily learn. Are we to understand ‘beyond’ in this context as a deeper or amplified kind of CCK? And what if the educational intention was to extend and amplify the knowledge of a topic, with the result that these defining features now formed part of CCK? Is this form of knowing content separate from the way mathematicians usually know mathematics or is some kind of intention required, and hence knowledge of teaching/learning to be so? What benefits are there to separating out mathematical knowledge in this way?

Defining SCK in this way raises the difficulty of clearly demarking what can be considered common knowledge from specialised knowledge. The point at which one shades into the other depends on various factors ranging from general considerations (educational level, the school system) to more specific ones (the teacher’s particular intentions).

EXAMPLES OF SCK

By way of illustrating what SCK refers to, in what contexts it is used, and how it is applied, we collated various examples from the literature. These include classroom sequences, or episodes, in particular those in which the teacher has to deal with difficult or unexpected circumstances, which show how the teacher interacts with mathematics.

In this section we offer a full analysis of two of the most representative examples that have been employed to illustrate SCK. The examples are reported in the literature as specific educational tasks. So as to identify as explicitly as possible the purely mathematical features required to solve these tasks, we will go through each task step by step, unpacking the information.

In the first example (Figure 1), a subtraction problem is given along with a typical algorithm for solving it and two potential errors that pupils might make. The mathematical knowledge involved in the analysis of procedures leading to the detection of these errors (one of a teacher's specific task) is identified as SCK.

Subtraction by regrouping

The subtraction is presented:

```
  307
- 168
```

Most readers will know an algorithm to produce the answer 139, such as the following:

```
  2 9
  307
- 168
-----
  139
```

Many third graders struggle with the subtraction algorithm, often making errors. One common error is the following:

```
  307
- 168
-----
  261
```

Consider another error that teachers may confront when teaching this subtraction problem:

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  307
- 168
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  169
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Figure 1: Subtraction by regrouping. (Ball et al., 2008, pp. 396-397)

The authors offer the following commentary:

> …in the subtraction example […], recognizing a wrong answer is common content knowledge (CCK), whereas sizing up the nature of an error, especially an unfamiliar error, typically requires nimbleness in thinking about numbers, attention to patterns, and flexible thinking about meaning in ways that are distinctive of specialized content knowledge (SCK). In contrast, familiarity with common errors

> and deciding which of several errors students are most likely to make are examples of knowledge of content and students (KCS). (pp. 401)

This example offers a description of SCK which leads us to wonder about the terms employed: What does ‘sizing up the nature of an error’ refer to? What is meant by an ‘unfamiliar error’? What does it mean to have ‘nimbleness in thinking about numbers’ or ‘flexible thinking about meaning’? What can we say about going beyond the answer to the subtraction? Indeed, we could ask ourselves what can be identified as purely mathematical here?

To answer this last question, let us go through the mathematical arguments leading to the identification and characterisation of the origin of each of the errors set out in the example.

Regarding the first, the algorithm is misapplied such that the smaller number is subtracted from the larger one in each of the three columns, resulting in error as the pupil fails to grasp the importance of the relationship between the top and bottom rows in the subtraction, as Ball *et al* (ibid) make clear. The way of thinking which leads to this error is the understanding of subtraction as the ‘distance’ between two numbers: the fact that ‘1’ appears at the foot of the column with ‘7’ and ‘8’, ‘6’ at the foot of the column with ‘0’ and ‘6’, and ‘2’ at the foot of the column with ‘3’ and ‘1’ strongly suggests that this was the operation applied to these numbers.

In the second example, the source that Ball *et al* (ibid) suggest for the error is a failure to recognise the positional values of the numbers at the moment of regrouping. To this we can add the consideration that according to the algorithm, zero is associated with an absence of value, so that it cannot *lend*, forcing it to borrow from the ‘3’. The knowledge which leads to this interpretation is the *statements* about 0 and the use of the subtraction algorithm.

In both examples, ample understanding is required about the mechanics of subtraction by regrouping works, and particularly the expanded notation of numbers.

According to the reasoning of the authors, we can say that the kind of knowledge identified above (the use and justification of the subtraction algorithm, numerical notation, subtraction as the ‘distance’ between two numbers, the statement about zero) would fall within SCK, given that it refers to knowledge put to use by the teacher in providing or evaluating mathematical explanations of how such errors occur, in addition to recognising and analysing them. However, we consider that the mathematical elements brought to light in the detailed analysis of the task do not provide sufficient evidence to guarantee that such knowledge is exclusive to mathematics teachers. What is more, all the knowledge involved could be categorised as Common Content Knowledge, depending on the researcher’s beliefs regarding how the pupil should understand the topic in question.

This example allows us to see one of the most repeated difficulties in the literature, the lack of a clear distinction between CCK and SCK.

Another example which is a particular challenge for mathematics teachers consists in coming up with a story which represents the division of fractions, such as $1\frac{3}{4} \div \frac{1}{2}$. We will focus on the second part of the activity suggested by Suzuka *et al* (2009), in which the teacher has to analyse a story which apparently contains errors in the set up, but which has the correct answer (Figure 2).

Story Problem

I have two pizzas. My friend eats one quarter of one of the pizzas and then I split what's left evenly between two of my other friends, Dan and Heather, and each person gets three and a half pieces of pizza.

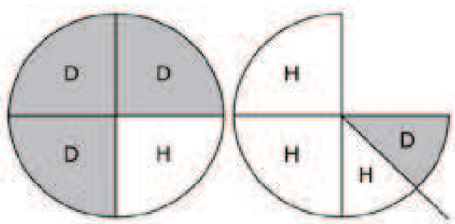

Figure 2: Incorrect story problem to represent $1\frac{3}{4} \div \frac{1}{2}$. (Suzuka et al., 2009, p. 11)

As mentioned above, one of the skills considered representative of SCK is that of interpreting mathematical productions – whether by pupils, other teachers or written material – something that this example embodies.

Let us repeat the exercise of going over the task making use of purely mathematical arguments to identify and describe the specific knowledge required to solve it. For this example we set ourselves the task of identifying the knowledge which enables one to know, on the one hand, that the result is correct, and on the other, that the setup of the story is incorrect.

Understanding why the result is correct requires being able to see the relation between the operation that the task intends to represent $\left(1\frac{3}{4} \div \frac{1}{2}\right)$ and the operation actually represented by the story problem $(7 \div 2)$. In the first of these, given that the divisor is a fraction, it is not possible to devise a natural context based on the partitive sense of division. The metamorphosis into the second operation attempts to express precisely this sense of division, but in this formulation it is not $1\frac{3}{4}$ pizzas that are divided but 7 slices of pizza (the size of each of which is $\frac{1}{4}$). This number of slices is the numerator of the improper fraction deriving from the mixed number: $1\frac{3}{4}=\frac{7}{4}$ The dividend in the second operation is the numerator of the corresponding fraction, once this has been transformed into an equivalent so that the denominators of both are the same: $\frac{1}{2}=\frac{2}{4}$ That is:

$$1\frac{3}{4}\div\frac{1}{2}=\frac{7}{4}\div\frac{2}{4}=\frac{7\times 4}{2\times 4}=\frac{7}{2}$$

It should be noted that we are not suggesting that this was the mental process by which the story problem was devised, rather we are setting out the mathematical arguments which allow one to analyse and account for the equivalence between the operations and for the answer being the same in both cases.

Hence, the knowledge involved in this first part is: knowing that the quotient of two fractions is equal to the quotient of any two equivalent fractions; knowing an algorithm for dividing fractions; knowing the multiplicative inverse property of numbers.

In order to understand why the problem posed in the story is incorrect, the required mathematical knowledge concerns the use of the meanings of division $a\div b$ as quantifier (How many times does b go into a?) and as sharing out (How many does each b get if we share out a?). The meaning inherent in the story problem cannot be extrapolated to the operation to be represented.

As with the example of subtraction, there is no purely mathematical knowledge here that can be seen as exclusive to mathematics teachers, or to which the pupil cannot have access.

CONCLUSION

In this paper we have aimed to scrutinise elements of knowledge pertaining to SCK. We began with an analysis of definitions and looked closely at examples which, according to the authors, involved specialised knowledge. We kept in mind throughout the notion of SCK as purely mathematical knowledge, whether viewed as an accumulation of special knowledge or as a special way of regarding content.

The definitions review we carried out leads us to conclude that these always employ elements which are extrinsic to specialised knowledge, such as making reference to other professions or the notion of going beyond CCK. In the analysis of the examples, we found that the specific tasks called for knowledge of meanings, properties and definitions of the mathematical topics involved. Nevertheless, it is natural to ask whether this knowledge can be considered specialised with respect to mathematics teachers. We would maintain that the answer is ‘no’. We cannot see any way of viewing the topic concerned that is particularly special, nor can we perceive any specialist mathematical knowledge which is habitually inaccessible to pupils or other professionals. What is evident is a specific use of this knowledge.

We are not trying to say that mathematics teachers' specialised knowledge does not exist; however, the data suggest that it might not be exclusive to the mathematical domain. We believe that it is impossible to think about this kind of knowledge without bringing to mind knowledge about mathematics teaching, such as ways of constructing the subject, the development of complexity within topics, and the features of learning mathematical content, amongst others. Specialised knowledge takes into account more aspects than meanings, properties and definitions.

The above conclusions form part of a series of considerations and reflections which together have led the research group SIDM in the Department of Mathematics Teaching at the University of Huelva (Spain) to work towards the development of a model which focuses on the study of what is specialised in terms of the results of an interaction between types of knowledge of and about mathematics, the structure, the teaching, the characteristics and standards of mathematics education, as well as connections to beliefs about mathematics (and its teaching/learning), mathematical knowledge always occupying the central focus. This work is the first of a series of papers (Carreño, Rojas, Montes, & Flores, 2012; Carrillo, Climent, Contreras, & Muñóz-Catalán, 2012; Montes, Aguilar, Carrillo, & Muñóz-Catalán, 2012) to be presented in this volume with the aim of offering a full picture of our advances as a research group (Carrillo *et al*, 2012, in this volume).

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