Abstract—The configuration of a series active power filter (APF) and a parallel passive filter (PPF) has proven to be an efficient system for nonlinear load compensation. For this topology, different compensation strategies have been proposed to control the series APF. The most effective strategy determines the APF reference voltage as a proportion of the source current harmonics. The proportionality constant in the control algorithm implementation is related to the APF gain and the system dynamics. In this paper, the system state model has been obtained for three control strategies of series APF: voltage proportional to source current harmonics, voltage opposite to the load voltage harmonics and a hybrid strategy which combines both previous. The resulting model analyses provide the information needed to establish design criteria for each strategy, both in terms of harmonic filtering and the system stability. The three strategies were compared in two different situations: sinusoidal supply voltages and distorted supply voltages. Finally, results of an experimental prototype developed for this purpose allowed the proposed analysis to be verified.

Key words
Power quality, active filters, hybrid filters, harmonics, state model.

1. Introduction
Active power filters (APF) have shown to be an effective technology to eliminate harmonics and to compensate nonlinear loads [1-3]. The shunt connection has been the most studied topology, where the APF is connected in parallel with the load. One of its traditional uses is the elimination of current harmonics produced by loads which generates such disturbances, this is HCS loads (Harmonic Current Source) [4,5]. However, the parallel APF is not suitable in situations where the load generates voltage harmonics, HVS loads (Harmonic Voltage Source). In this case, series connection APF configuration has been proposed and different control strategies have been tried out [6]. Compensation systems composed only of an APF either shunt or series connection do not solve completely the problem of harmonic elimination for all load types. So, other configurations have been proposed [1]. They combine series and parallel topologies, active and passive filters.

Among the hybrid topologies, the series APF with parallel passive filter (SAPPF) is the most studied. Specifically for this configuration, different control strategies have been proposed. One of them has its origin in the early 90s; the control objective was based on generating a voltage proportional to the source current harmonics. This APF control strategy allowed to improve the filtering features of parallel connection passive filter [7]. The functionality of the new strategy is analyzed from a steady state model; however, this theoretical development is not suitable to study the system stability or to determine the proportionality constant value.

Subsequently, other control strategies for the series APF appeared. Thus, in [8-10] three control strategies are analyzed.
The resulting model allows to establish the control specifications to ensure system stability and to eliminate the source current harmonics. For this purpose a simulation platform based on MATLAB-Simulink has been developed. Each strategy is analyzed in the cases of sinusoidal supply voltage and nonsinusoidal supply voltage.

Finally, a SAPPF experimental prototype has been developed. The three compensating strategies have been applied to the series APF, which has allowed the theoretical analysis to be contrasted.

## 2. State Model of Hybrid Active Power Filter

Fig. 1 shows a three-phase circuit with a voltage source that feeds a HCS type nonlinear load and a SAPPF hybrid filter. The source impedance is constituted by a resistance $R_S$ and an inductance $L_S$. The active power filter is connected in parallel with the load. It consists of two LC branches tuned to $5^{th}$ and $7^{th}$ current harmonics. The passive filter is connected in series with the source through a coupling transformer. The source current harmonics. For this purpose a theoretical analysis to be contrasted.

The circuit used to obtain the system state model is shown in Fig. 2. This circuit represents the equivalent single-phase network of Fig. 1 for any $h$ order harmonic different of the fundamental harmonic. The active filter is modeled by a controlled voltage source, $u$. As load model the Norton equivalent has been used, defined by a resistor, $R_L$, in parallel with an inductance, $L_L$, and a current source $i_{LL}$. This current source will be zero for the fundamental harmonic. The active filter is modeled by a controlled voltage source, $u$. As load model the Norton equivalent has been used, defined by a resistor, $R_L$, in parallel with an inductance, $L_L$, and a current source $i_{LL}$. This current source will be zero for the fundamental harmonic. The passive filter consists of two LC branches tuned to $5^{th}$ and $7^{th}$ current harmonics.

![Fig. 2. System with SAPPF filter and load type HCS](image)

In this case the $B_1$ vector has the form

$$B_1 = \begin{bmatrix} \frac{1}{L_S} & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

(4)

$B_2$ is defined by the matrix

$$B_2 = \begin{bmatrix} \frac{1}{L_S} & 0 & 0 & 0 & 0 \\ \frac{R_L}{L_S} & -\frac{R_L}{L_S} & \frac{R_L}{L_S} & -\frac{R_L}{L_S} & 0 \\ \frac{R_L}{L_7} & -\frac{R_L}{L_7} & \frac{R_L}{L_7} & -\frac{R_L}{L_7} & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(5)

And the input vector takes the form

$$v = \begin{bmatrix} v_S \\ i_{LL} \end{bmatrix}^T$$

(6)

The matrices $C$, $D_1$ and $D_2$ depend on the selected output. If the source current ($i_S$) is taken as an output signal, the matrices are as follows

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(7)
3. Analysis of Different Compensation Strategies

In this section the state equations corresponding to the strategies SCD, LVD and CVD are obtained. The different developments allow the influence of each control strategy on the system matrix to be analyzed.

A. Control strategy of source current detection

When the control strategy proportional to the source current harmonics is applied, the voltage generated by the APF is given by the expression

\[ u = k_i_s \]  \hspace{1cm} (9)

Where \( k \) is the proportionality constant. In matrix form, it can be expressed as

\[
\[ \begin{bmatrix}
  k & 0 & 0 & 0 & 0 \\
  0 & k & 0 & 0 & 0 \\
  0 & 0 & k & 0 & 0 \\
  0 & 0 & 0 & k & 0 \\
  0 & 0 & 0 & 0 & k 
\end{bmatrix}
\]  \hspace{1cm} (10)

Therefore, this strategy is based on the feedback of the state vector through a gain matrix, \( K \), which is defined by

\[
K = \begin{bmatrix}
  k & 0 & 0 & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (11)

With this control strategy, the state equation (1) can be expressed as

\[
\dot{x} = (A + B_1 K_1) x + B_1 v
\]  \hspace{1cm} (12)

Where the system matrix is defined by

\[
A' = (A + B_1 K_1) = \begin{bmatrix}
  \frac{R_i + L_s k_i_s}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & 0 & 0 \\
  \frac{R_i}{L_s} & -\frac{R_i + L_s k_i_s}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & 0 \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + L_s k_i_s}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + L_s k_i_s}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  0 & \frac{1}{C_i} & 0 & 0 & 0 & 0 \\
  0 & 0 & \frac{1}{C_i} & 0 & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (13)

By comparing (3) and (13), only the first term of the \( A' \) matrix is different from the \( A \) matrix with this strategy.

B. Control strategy of load voltage detection

In this control strategy the voltage generated by the active filter must generate a voltage waveform defined by means of

\[
u = -k_i v_L \]  \hspace{1cm} (14)

Where \( k \) models the instrumentation sensitivity and \( v_L \) is the voltage at the load side. In Fig. 2 circuit, the voltage \( v_L \) gets the next expression

\[
v_L = R_L i_s - R_L i_s - R_L i_s - R_L i_s - R_L i_s \]  \hspace{1cm} (15)

According to (14) and (15) the control signal can be expressed by

\[
u = -k_i R_L i_s + k_i R_L i_s + k_i R_L i_s + k_i R_L i_s + k_i R_L i_s \]  \hspace{1cm} (16)

Thus, the system input vector is given by

\[
u = K_1 x + K_2 v
\]  \hspace{1cm} (17)

Where

\[
K_1 = \begin{bmatrix}
  -k_i R_L & k_i R_L & k_i R_L & k_i R_L & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (18)

And

\[
K_2 = \begin{bmatrix}
  0 & k_i R_L 
\end{bmatrix}
\]  \hspace{1cm} (19)

Finally, the state equation can be written as follows

\[
\dot{x} = (A + B_1 K_1) x + (B_2 + B_1 K_2) v
\]  \hspace{1cm} (20)

Now, the system matrix is

\[
A' = (A + B_1 K_1) = \begin{bmatrix}
  \frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & 0 \\
  \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  0 & \frac{1}{C_i} & 0 & 0 & 0 & 0 \\
  0 & 0 & \frac{1}{C_i} & 0 & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (21)

This strategy modifies the first row of the system matrix and the matrix that multiplies the input vector.

C. Control strategy of current and voltage detection

With the hybrid strategy, the active filter must generate a voltage waveform defined by means of

\[
u = k_i v_s - k_i v_L \]  \hspace{1cm} (22)

According to Fig. 2 circuit, the control signal can be calculated by

\[
u = (k_i - k_i R_L) i_s + k_i R_L i_s + k_i R_L i_s + k_i R_L i_s + k_i R_L i_s \]  \hspace{1cm} (23)

Which is expressed in matrix form as follows

\[
u = K_1 x + K_2 v
\]  \hspace{1cm} (24)

Where

\[
K_1 = \begin{bmatrix}
  (k_i - k_i R_L) & k_i R_L & k_i R_L & k_i R_L & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (25)

And

\[
K_2 = \begin{bmatrix}
  0 & k_i R_L 
\end{bmatrix}
\]  \hspace{1cm} (26)

So, the state equation is defined by

\[
\dot{x} = (A + B_1 K_1) x + (B_2 + B_1 K_2) v
\]  \hspace{1cm} (27)

Here the system matrix is as follows

\[
A' = (A + B_1 K_1) = \begin{bmatrix}
  \frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  \frac{R_i}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} & -\frac{R_i + R_L (1-k_i)}{L_s} & \frac{R_i}{L_s} & \frac{R_i}{L_s} \\
  0 & \frac{1}{C_i} & 0 & 0 & 0 & 0 \\
  0 & 0 & \frac{1}{C_i} & 0 & 0 & 0 
\end{bmatrix}
\]  \hspace{1cm} (28)

This strategy modifies the first row of the system matrix and the matrix that multiplies the input vector.
The CVD control strategy modifies the first row of the matrix and the matrix system that multiplies the input vector.

4. Experimental Results

An experimental prototype of the circuit shown in Fig. 1 has been implemented. The system is supplied by a programmable three-phase source, the California Instruments 4500-L model. The source generates a balanced sinusoidal waveform, 50 Hz non-sinusoidal voltage source. In the first test the voltage source is connected at the inverter dc side to maintain absolute value about 0.03) implies that the 5th harmonic drops to a 3%. Therefore, if the supply voltage is distorted more clearly. It shows the gain at a frequency of 250 Hz harmonic content. Regardless of the chosen input signal or the output variable, a higher k value becomes smaller the Bode gain. However, the increase of k is not proportional to the decrease of the Bode gain. Fig. 3, reflects this fact more clearly. It shows the gain at a frequency of 250 Hz (5th harmonic) for different k values. A gain of -30 dB (in absolute value about 0.03) implies that the 5th harmonic drops to a 3%. Therefore, if the supply voltage is distorted a value of k=5 would be enough. However, considering that the distortion source is the load current a k proportionality constant is fixed at 50. This will guarantee the system stability and will reduce the distortion at least until -30 dB.

On the other hand, from the point of view of eliminating harmonics it is convenient to establish the best k value. The aim is that the system presents a low gain for a given harmonic content. Regardless of the chosen input signal or the output variable, a higher k value becomes smaller the Bode gain. However, the increase of k is not proportional to the decrease of the Bode gain. Fig. 3, reflects this fact more clearly. It shows the gain at a frequency of 250 Hz (5th harmonic) for different k values. A gain of -30 dB (in absolute value about 0.03) implies that the 5th harmonic drops to a 3%. Therefore, if the supply voltage is distorted a value of k=5 would be enough. However, considering that the distortion source is the load current a k proportionality constant is fixed at 50. This will guarantee the system stability and will reduce the distortion at least until -30 dB.

Table II. Poles situation for different strategies: (1) Without active filter; (2) SCD strategy; (3) LVD strategy; (4) CVD strategy.

Both waveforms are characterized by odd harmonics, except the 3 order and its multiples which are practically zero. The voltage rms value is 86.8 V and the current rms value 7.8 A. The most significant harmonics are the 5th and 7th. The active, reactive and apparent powers per phase consumed by the load plus passive filter are: P=0.71 kW, Q=0.07 kvar and S=0.71 kVA. The power factor improves until 0.99. There is a slight increase in the active power, which is justified by the resistive effect in the LC branches coils. Reactive power is also reduced because the LC branches is capacitive at the fundamental frequency. In what follows the passive and active filters are connected. In the first test with active filter, the strategy of source current detection will be applied. This strategy causes an alteration in the location of the uncompensated system poles; it is shown in the second row of Table II. Thus, the poles move away from the imaginary axis and the system stability is strengthened. On the other hand, a stability analysis shows that system is stable for those values of k>-1.8.

Fig. 3. Bode gain values versus k at frequency 250 Hz

Fig. 4c shows the source current and voltage waveforms at the point of common coupling. The voltage THD is 2.0%
and current THD is 2.3%, (Table III). There is a significative reduction in the THD with respect to the system without active filter. The voltage and current rms values are 87.6 V and 87.9 A respectively. On the other hand, the active, reactive and apparent powers per phase are \( P = 0.69 \, \text{kW}, \, Q = 0.03 \, \text{kvar} \) and \( S = 0.70 \, \text{kVA} \) and the power factor is 0.99.

In the following test the strategy of load voltage detection is applied to active filter. Taking into account (21), this strategy changes the system matrix and thus the location of the poles. Its behavior depends on the parameter \( k_v \), which models the instrumentation sensitivity. The value of \( k_v \) which becomes the unstable system is obtained by simulation. This occurs when \( k_v \geq 1.198 \), since the system would present a pole in the right half plane.

In the second test the SAPPF filter is connected to a distorted supply voltage. The 4500-iL source is programmed to generate a 5th harmonic of 12% of the fundamental harmonic. Fig. 5 shows the source current and the voltage at the point of common coupling waveforms before and after connecting the active filter. To fix the \( k \) values the results obtained in previous analysis are taken into account. Thus, the SCD strategy the \( k \) proportionality constant is 50 and using the CVD strategy is \( k = 29 \).

From the point of view of eliminating the harmonics the aim is that the Bode gain of the system is minimal for a given input signal. This condition is given for \( k_v = 1 \), when the current source of the load Norton equivalent is considered as input signal. When the supply voltage is distorted the active filter can even increase the source current harmonic distortion.

From another point of view, when the design specification is fixed to -30 dB for the Bode gain at low frequencies and \( k_v = 0.95 \) is considered, the \( k \) values must be 29. This \( k \) value ensures a maximum Bode gain of -30 dB, regardless of the considered distortion source. On the other side, when the supply voltage is slightly distorted, \( k \) can be reduced to 12, having a maximum gain of -30 dB at low frequencies. In this case the proportionality constant is fixed at \( k = 12 \).

Finally, the strategy of current and voltage detection has been applied. Regarding the system stability two parameters must take into account: \( k_v \) and \( k \). In the first analysis, it is considered that \( k_v = 1 \), which determines the minimum \( k \) value to make the system stable. This occurs for \( k > -1.8 \). This is the same result obtained with the SCD strategy. When \( k_v \) is 1.2, the system is stable if \( k > 0 \). Therefore, with the CVD strategy is achieved to provide greater robustness to the system, making it less sensitive to \( k_v \) variations. It is evident that for \( k_v < 1 \), the stability limit is found for values of \( k < -1.8 \). From the point of view of stability, if \( k_v \) has a value close to the unity the system will be stable when \( k \) is greater than zero, which will be two design specifications.

From the point of view of eliminating harmonics, when the supply voltage is sinusoidal the results are similar for the three compensation strategies. In the second test the SAPPF filter is connected to a distorted supply voltage. The 4500-iL source is programmed to generate a 5th harmonic of 12% of the fundamental harmonic. Fig. 5 shows the source current and the voltage at the point of common coupling waveforms before and after connecting the active filter. To fix the \( k \) values the results obtained in previous analysis are taken into account. Thus, the SCD strategy the \( k \) proportionality constant is 50 and using the CVD strategy is \( k = 29 \).
Table IV summarizes the most significative measured values: THD, harmonics, power and power factor. The SCD and CVD strategies achieve the source current THD most reduced. In any case, the proposed design methodologies have allowed the source current distortion to be minimized using each control strategy.

5. Conclusions
A system dynamic analysis has been carried out by means of the state equations. It has allowed the design guidelines for the APF control to be established. Specifically, using the CVD compensation strategy, the APF voltage is obtained by a combination of a term proportional to the source current harmonics and another term in opposite to the load voltage harmonics. This requires the specification of the proportionality constant in the first term and the sensitivity of the instrumentation in the second term. In this paper, the analysis has allowed the values of these parameters to be chosen from a reduction in harmonic distortion of source current specified by design. Finally, the proposed methodology has been applied to an experimental SAPPF using each of the three mentioned compensation strategies. The voltage and current waveforms and the measured results in terms of distortion and power before and after compensation have been presented.

Table IV. Experimental results of the SAPPF filter with nonsinusoidal source. (1) Without filter; (2) SCD strategy; (3) LVD strategy; (4) CVD strategy

References