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Three-gluon running coupling from lattice QCD at $N_f = 2 + 1 + 1$: a consistency check of the OPE approach

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Abstract: We present a lattice calculation of the renormalized running coupling constant in symmetric (MOM) and asymmetric ($\widetilde{\text{MOM}}$) momentum substraction schemes including u, d, s and c quarks in the sea. An Operator Product Expansion dominated by the dimension-two $\langle A^2 \rangle$ condensate is used to fit the running of the coupling. We argue that the agreement in the predicted $\langle A^2 \rangle$ condensate for both schemes is a strong support for the validity of the OPE approach and the effect of this non-gauge invariant condensate over the running of the strong coupling.

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Contents

The running of QCD coupling constant is one of the key ingredient for the confrontation of the experimental results to the perturbative expressions. The value of $\alpha_{\rm QCD}$ at a given scale or alternatively the parameter $\Lambda_{\rm QCD}$, which controls the perturbative running, can be extracted from experimental results. The running coupling constant can also be computed from lattice QCD calculations by a large variety of methods. Among the most extensively applied one could find the implemention of the Schrödinger functional scheme (see, for instance, [1–4] and references therein), those based on the perturbative analysis of short-distance sensitive lattice observables as the inter-quark static potential (see for instance [5, 6]), the "boosted" lattice coupling (see [7–10] and references therein), moments of charmonia two-point correlation functions (see [11–13] and references therein) or, in particular, those based on the study of the momentum behaviour of Green functions (see [14–20], for instance). The latter, in particular, need for a coupling to be nonperturbatively defined in a MOM-type scheme by fixing the QCD propagators (two-point Green functions) and one particular three-point Green-function for a chosen kinematical configuration to take, after renormalization, their tree-level result at the renormalization scale.

The analysis of the running for a so-defined coupling at intermediate energies, roughly from 3 to 10 GeV, deserves great interest as it provides with a privileged room for the confrontation of lattice nonperturbative results to perturbation theory, at any order, where the nature and impact of nonperturbative corrections can be studied (see ref. [21] for a recent review). For instance, the study of the coupling defined from the asymmetric ($\widetilde{\rm MOM}$) three-gluon vertex and estimated from quenched lattice data revealed the main role of nonperturbative power corrections to account for its running [16]. Then, in a vast series of papers [17, 18, 22–30], some of us have exhaustively proven that Wilson's Operator Product Expansion (OPE) provides a general framework to include non-perturbative contributions, and that its application to QCD couplings for several renormalization schemes allows a coherent and simple explanation of the running obtained from the lattice for momenta as low as $\sim 2-3$GeV. The leading OPE contribution has been shown to result from the non-vanishing condensate of the gauge-dependent dimension-two local operator A^2 [31], which, in the last decade, received profuse attention within the context of the so-called *refined Gribov-Zwanziger* approach [32–34] but also in many others (see [35–44]).

Among the MOM schemes that have been studied, that defined for the ghost-gluon vertex with zero incoming ghost momentum (called T-scheme) has been extensively exploited in the last few years, due mainly to a well-known Taylor's result [45] whereby the proper ghost-gluon vertex renormalization constant for this scheme is proven to be exactly one in Landau gauge. Thus, the MOM T-scheme coupling can be computed only from ghost and gluon propagators, without involving a three-point function. The latter allows for a very precise determination of α_s in a range of momenta which makes possible to get an accurate estimate of $\Lambda_{\rm QCD}$ that, for realistic unquenched lattice simulations, succesfully compares to its value from experiments [28, 29, 46].

In any other scheme, the renormalized coupling requires the lattice evaluation of a vertex function, i.e., a three point correlation function. This is the case for the coupling defined from the three-gluon vertex,[1] where one can cook out as many different renormalization schemes as there are possible kinematical configurations. As the signal for a three-point correlation function, suffering from stronger statistical fluctuations, is much harder to be extracted from lattice simulations than the one for two-point functions, the precision so attained is not comparable with the one achieved when using the T-scheme coupling. The interest of computing α_s in different schemes is therefore not to obtain a precise value of $\Lambda_{\rm QCD}$ but, rather, to test the OPE framework and to gain thus some insight into the nature of the nonperturbative corrections.

In this paper we present the lattice evaluation of the MOM QCD coupling defined through the three gluon vertex for two different kinematical configurations: the symmetric (three equal momenta) and asymmetric (one vanishing momentum) ones. The high-statistics ensemble (800 configurations) of lattice gauge fields we exploit takes into account the dynamical generation of up, down, strange and charm quarks ($N_f = 2+1+1$). This leaves us with two main "*aces*" for our game: (i) the Wilson coefficients for the leading contribution in the OPE of the two couplings, as will be seen, differ very much from each other; and (ii) the perturbative running is very reliably known as the same $N_f = 2+1+1$ lattice configurations provides, *via* the T-scheme coupling determination, with an accurate estimate of $\Lambda_{\rm QCD}$ [46], compatible with PDG world average [50], that can be used here. We put ourselves in a near unbeatable position to check the OPE framework, as the nonperturbative contributions supplementing the perturbative running to account for the lattice data of both couplings can be properly isolated and compare to each other. One can see then if they differ as much as OPE predicts.

The structure of the paper is as follows: in section 1 the renormalization schemes and lattice setup used are described. In section 2 a reminder of the OPE results has been included. Finally in section 3 our main results are presented and we concluded in section 4.

[1]The three-gluon vertex has been also the object of a recent study [47] grounding a QCD effective charge definition within the framework of the background field method and the pinching technique [48, 49].

1 Lattice data and renormalization schemes

The starting point for this calculation shall be the gauge configurations produced by the European Twisted Mass collaboration (ETMC) for $2+1+1$ dynamical quark flavors that provide a realistic description of the QCD dynamics including heavy flavours. These gauge field configurations, after fixing Landau gauge, allow to compute the renormalized running coupling in momentum substraction schemes. In particular we will focus on the coupling defined from three-gluon vertices.

One question in order to be discussed here, before dealing with the computation of the three-gluon vertex, is the problem of the Gribov ambiguity [51] which is suffered by the gauge-dependent Green functions as the gluon propagator or the three-gluon correlation function. The standard Landau-gauge fixing procedure by the minimisation of a functional of the gauge field, A_μ^a, verifying $\partial_\mu A_\mu^a = 0$ with the Fadeev-Popov operator being positive, leads to many local minima of the gauge orbit, usually called "Gribov copies". Such an ambiguity on the gauge-fixing may introduce disrupting deviations for the confrontation of continuum and Landau-gauge lattice quantities. On the lattice, this ambiguity has been scrutinized by comparing the results from a "best copy", selected as the minimum of the functional for a sample of random copies, with the ones from the "first copy" resulting from the minimisation [52–54]. The selection of the "best copy" has been also improved in recent investigations by the application of the so-called simulated annealing (SA) gauge-fixing algorithm [55–57]. The main conclusion from these investigations is that Gribov-copy effects are found not to have any impact on two-point SU(2) Green functions above a given momentum, $p_{\rm min}$. This momentum $p_{\rm min}$ is also found to decrease with the lattice size in physical units, L, for a hypercubic lattice. The authors of ref. [57] studied results from simulations with L roughly ranging from 1 to 8 fm and generally concluded that Gribov-copy effects were relevant for $p < 1\,\text{GeV}$. In particular, their figure 5 shows that $p_{\rm min} \simeq 0.7\,\text{GeV}$ for $L \simeq 5\,\text{fm}$. In a recent paper [58] we have also analyzed two-point SU(3) gluon and ghost correlators, where we also used the gauge fields exploited here, and found[2] no indication of Gribov-copies ambiguities for momenta roughly above 1.7 GeV, and described the running with momenta with a continuum prediction incorporating four-loop perturbation theory and OPE power corrections. We deal here with SU(3) gauge fields simulated over a lattice with a spatial size of 3 fm, and study the momentum behaviour of its three-point correlation function, roughly above 3 GeV, where it can reasonably expected to be safe from the Gribov-ambiguity problem.

The three-gluon vertex (figure 1) can be computed from the lattice for any momenta p_1, p_2 and p_3 satisfying $p_1+p_2+p_3=0$. In particular, we will concentrate on the symmetric three gluon vertex ($p_1^2=p_2^2=p_3^2$) and the asymmetric one ($p_3=0$ and therefore $p_2=-p_1$). The renormalized coupling can be straightrowardly defined from gluon propagators and vertices (a detailed description of the procedure can be found in [15]). The renormalized

[2] In particular, we found a perfect matching of SU(3) Taylor coupling results from many different non-hypercubic lattice simulations, where the spatial size roughly ranges from 2 to 3 fm and used the first-copy gauge-fixing approach for all of them, over a fitting window with very UV momenta, free of ambiguities, and IR momenta above 1.7 GeV

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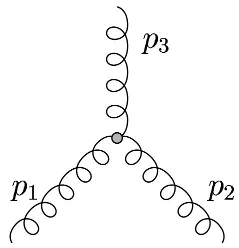

Figure 1. Three gluon vertex. MOM renormalization prescription implies that full three point function behaves as a renormalized coupling (grey circle) times the three outgoing renormalized propagators.

coupling is defined by:

$$g_R(\mu^2) = \frac{Z_3^{3/2}(\mu^2)G^{(3)}(p_1^2,p_2^2,p_3^2)}{\left(G^{(2)}(p_1^2)G^{(2)}(p_2^2)G^{(2)}(p_3^2)\right)^{1/2}} \tag{1.1}$$

where μ^2 is the renormalization scale, to be fixed for each renormalization scheme, $G^2(p^2)$ is the bare gluon propagator extracted from the lattice:

$$G^{(2)}(p^2) = \frac{\delta_{ab}g^{\mu\nu}}{3(N_C^2-1)}\langle \tilde{A}_\mu^a(p)\tilde{A}_\nu^b(-p)\rangle\,, \tag{1.2}$$

$Z_3(\mu^2)=\mu^2G^{(2)}(\mu^2)$ is the gluon field renormalization constant, and $G^{(3)}(p_1^2,p_2^2,p_3^2)$ is the scalar function extracted from three gluon vertex $G^{(3)}{}^{abc}_{\mu\nu\rho}(p_1,p_2,p_3)=\langle\tilde{A}_\mu^a(p_1)\tilde{A}_\nu^b(p_2)\tilde{A}_\rho^c(p_3)\rangle$ This scalar function is defined as the coefficient of the tree level tensor and is obtained after projecting the vertex onto the adequate tensor as described in [15].

This procedure allows to compute the running coupling $\alpha(\mu^2) = \frac{g_R^2(\mu^2)}{4\pi}$ in momentum substraction schemes both from the symmetric three gluon vertex (MOM) and from the asymmetric one ($\widetilde{\text{MOM}}$).

The symmetric vertex requires the three momenta p_1, p_2 and p_3 to satisfy the constrain $p_1+p_2+p_3=0$ simultaneously with $p_1^2=p_2^2=p_3^2$ which is rather rare in the lattice. It means that there are rather few momenta where the vertex can be evaluated. For the asymmetric one, the constrain is less restrictive and the vertex can be evaluated at any lattice momenta p.

As mentioned above, the two and three-point gluon Green functions will be computed from lattice gauge field configurations simulated at N_f=2+1+1 by the ETM collaboration [59, 60]. The details of the computation can be found in [28] and references therein. We have exploited here a set of 800 configurations for a lattice volume $48^3\times 96$ and $\beta=2.10$, where the lattice parameters $\kappa_c = 01563570$, $\mu_l = 0.002$, $\mu_\sigma = 0.120$, $\mu_c = 0.385$, are fixed so that light quark mass is set to $\sim 20\,\text{MeV}$ and the strange and charm are set to $\sim 95\,\text{MeV}$ and $1.5\,\text{GeV}$ respectively (see [28] and references therfein for the details of the simulations). According to ref. [46], the lattice spacing corresponding to this set-up is $a(2.10) = 0.0583(11)\,\text{fm}$.

Contrarily to the continuum ones, the lattice scalar functions do not depend only on the momentum squared p^2, due to the lattice discretization. Indeed, the lattice discretization

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breaks the $O(4)$-symmetry introducing the $O(4)$-breaking lattice artefacts. These artefacts can be efficiently removed by using the so-called $H(4)$-extrapolation procedure [61–63], which allows for the recovery of the $O(4)$-symmetric result for any Green function evaluated on the lattice. That procedure is basically grounded on the idea that, as the $O(4)$ rotational invariance is broken in the lattice down to the $H(4)$ isometry group, any correlator's scalar form factor evaluated on the lattice should be univocally determined by the knowledge of the four $H(4)$ invariants, $p^{[i]} = \sum_\mu p_\mu^i$, with $i = 2, 4, 6, 8$ and μ is a Lorentz index.

In its simplest form, the method works by fitting from the available lattice data the $O(4)$-symmetric result, which only depends on the first $H(4)$ (and $O(4)$) invariant $p^{[2]} \equiv p^2$, and the $O(4)$-breaking corrections that will be supposed to depend only on the two first invariants, p^2 and $p^{[4]}$, and that can be written down with the help of general dimensional arguments. To this purpose, the lattice data for all the different lattice momenta sharing the same two $H(4)$ invariants (belonging to the same $H(4)$ orbit) need to be averaged. Thus, the $H(4)$-extrapolation works efficiently for the gluon propagator and asymmetric vertex, where a large number of $H(4)$ orbits is available and where, in many cases, several orbits share the same $O(4)$ invariant, p^2. The procedure is not so efficient when not enough $H(4)$ orbits are available for a good fit, as happens for the symmetric vertex because of the lower number of lattice momenta at which it can be evaluated, due to the constrain that the three squared momenta entering the vertex must be the same. The method also becomes the less and less efficient for increasing momenta, where the other invariants, $p^{[6]}$ and $p^{[8]}$, cannot be properly neglected, that increasing the number of parameters to be fitted with regard to the number of available $H(4)$ orbits. When the latter happens, the data plotted in terms of momenta appear to show the characteristic oscillations that can be attributed to $O(4)$-breaking artifacts and the errors, also accounting for the $H(4)$ extrapolation, become larger (this can be seen in figure 2).

In practice, to avoid the noise induced by the non-properly-cured lattice artefacts introduces a limitation for the largest momenta that can be used for the fits. The upper bound for the fitting window needs to be consequently smaller for the schema MOM than for $\widetilde{\text{MOM}}$. In particular, as will be seen in the next section, we will take $a(2.10)p \simeq 1.6$ (around 5.5 GeV, in physical units) as the upper bound in the $\widetilde{\text{MOM}}$ case, and $a(2.10)p \simeq 1.3$ (around 4.5 GeV) in MOM.

2 OPE nonperturbative predictions

The running of the strong coupling constant with momentum, obtained from QCD perturbation theory corrected by a nonperturbative leading OPE power contribution, can rather generally read [26, 28]

$$\alpha_R(\mu^2) = \alpha_R^{\text{pert}}(\mu^2) \left(1 + \frac{c_R}{\mu^2} \left(\frac{\alpha_R^{\text{pert}}(\mu^2)}{\alpha_R^{\text{pert}}(q_0^2)} \right)^{1-\gamma_0^{A^2}/\beta_0} \right.$$
$$\left. \times R\left(\alpha_R^{\text{pert}}(\mu^2), \alpha_R^{\text{pert}}(q_0^2)\right) \frac{g_R^2(q_0^2)\langle A^2 \rangle_{R,q_0^2}}{4(N_C^2-1)} + o\left(\frac{1}{\mu^2}\right) \right) , \tag{2.1}$$

where the subindex R specifies any particular renormalization scheme and $\alpha^{\rm pert}$ gives the running behaviour perturbatively obtained from the integration of the QCD beta function at that R scheme,

$$\frac{d}{d\ln\mu^2} h_R \;=\; -\left(\beta_0 h_R^2 + \beta_1 h_R^3 + \beta_2^R h_R^4 + \dots\right) \tag{2.2}$$

with $h_R = \alpha_R(\mu^2)/(4\pi)$ and $\beta_0 = 11 - 2/3N_f$, $\beta_1 = 102 - 38/3N_f$, being scheme-independent coefficients. The result for $\alpha^{\rm pert}$ from the integration of eq. (2.2) and its conventional perturbative inversion, in terms of momenta and the QCD scale Λ_R, can be found in [50]. Within the bracket, c_R is given by the tree-level Wilson coefficient contribution, $\gamma_0^{A^2}$ is the first coefficient for the local operator A^2 anomalous dimension, determining the Wilson-coefficient leading-logarithm contribution,

$$1 - \gamma_0^{A^2}/\beta_0 = \frac{27}{132 - 8N_f}\,, \tag{2.3}$$

which is found to be scheme-independent; and $R(\alpha, \alpha_0)$ encodes the higher-order logarithmic corrections for the leading Wilson coefficient [30]. In refs. [26, 29, 46], eq. (2.1) particularized to the MOM T-scheme accounted very accurately for the running of the corresponding lattice data with momenta. This allowed for a precise determination of Λ_T, and hence $\Lambda_{\overline{\rm MS}}$ pretty in agreement with the PDG [50] "*world average*".

Hereupon, we will mainly concentrate on the running coupling renormalized in both MOM and $\widetilde{\rm MOM}$ schemes, for which the three-loop beta coefficients are known.[3] For $N_f = 4$, one is left with

$$\beta_2^{\widetilde{\rm MOM}} \;=\; 814.56\,, \qquad \beta_2^{\rm MOM} \;=\; 641.16\;; \tag{2.4}$$

and with the following ratios

$$\frac{\Lambda_{\overline{\rm MS}}}{\Lambda_{\widetilde{\rm MOM}}} \;=\; 0.443\,, \qquad \frac{\Lambda_{\overline{\rm MS}}}{\Lambda_{\rm MOM}} \;=\; 0.463\,, \tag{2.5}$$

that can be exactly obtained from the first non-trivial coefficient for the expansion of MOM and $\widetilde{\rm MOM}$ couplings in terms of the $\overline{\rm MS}$ one. The tree-level Wilson coefficients for the strong coupling in both schemes have been also studied [18, 22] and their computation gives

$$c_{\widetilde{\rm MOM}} = 3\,, \qquad c_{\rm MOM} = 9\;. \tag{2.6}$$

It is worthwhile to recall that, in the $\widetilde{\rm MOM}$ case, as a consequence of the soft gluon field in the three-gluon Green function defining the vertex, eq. (2.1) only results after the factorization of a leading higher-dimension condensate[4] in the OPE, induced by a vacuum insertion approximation [22] (the same vacuum insertion approximation have been proven

[3] The four-loop beta coefficient appears also to be known in the $\widetilde{\rm MOM}$ case.

[4] The anomalous dimension first coefficient for the lower dimension operator in the $\widetilde{\rm MOM}$-case OPE expansion have been proven to differ from that of A^2 only in a negligible way [22].

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to work for the OPE expansion of the ghost-ghost-gluon Green function [64]). The function $R(\alpha, \alpha_0)$ is related to the A^2 anomalous dimension but also depends on the scheme we used to define the coupling [30]. For the MOM T-scheme [25], as the coupling can be directly related to gluon and ghost propagators involving no three-point Green function, it has been computed at the $O(\alpha^4)$-order [26, 30]. Its computation is nevertheless cumbersome when dealing with three-gluon Green function is needed. Thus, in the following, we will take $R(\alpha, \alpha_0) = 1$ and will work at the leading-logarithm approximation.

3 Results

Then, eq. (2.1) can be fitted to the lattice data, with $g^2\langle A^2\rangle$ as a free parameter, for both MOM and $\widetilde{\text{MOM}}$ couplings, with their perturbative predictions obtained by the integration of the beta function with the coefficients given in eq. (2.4) and the ratios of Λ's in eq. (2.5). We will take $\Lambda_{\overline{\text{MS}}} = 314\,\text{MeV}$, as an input from ref. [46][5] where, as above mentioned, the MOM T-scheme coupling is computed from the lattice and confronted with eq. (2.1), properly particularized (ref. [46] upgrades the previous results of refs. [28, 29]). Figure 2 shows the lattice results for the running coupling constant and the best fits with eq. (2.1) and $g^2\langle A^2\rangle$ from table 1, in the MOM and $\widetilde{\text{MOM}}$ schemes.

3.1 Discussion of fitted results

The OPE approach allows for the expansion of the matrix element of any non-local operator in terms of local operators, that can be conveniently organized in a hierarchy by their momentum dimensions. Then, the terms of that OPE expansion for any Green function provide with nonperturbative corrections that, after the sum rules factorization (SVZ) [66, 67], appear coded as a coefficient to be computed in perturbation (Wilson coefficient) and the nonperturbative condensate of a local operator. This condensate can be interpreted as the *vacuum expectation*, in the non-trivial nonperturbative QCD vacuum, of that local operator.[6] A consistent picture implies therefore that, with the proper renormalization of the local operators, their condensates should be "universal" and take the same value in the OPE expansion for any different Green function. Thus, within the framework of OPE and SVZ sum-rules approach, the gluon condensate needs to take similar values in the OPE for the two couplings. The agreement in the values of the extracted condensates is therefore a strong indication of the validity of this approach, at least for a window of momenta not lying in the deep IR domain. As the nature of the OPE condensates is the object of a recent controversy [69, 70], It is worth to point out that our check is validating the SVZ "technology" to compute the condensates but does not tell necessarily anything about the nature of these condensates.

The values of the condensates obtained in this paper are sizably larger than the ones reported in [28, 29, 46] for the T-scheme. It is however important to note that, in the T-

[5] It should be noted that [46] applied now a very recent result for the lattice scale setting [65], which slightly differs from that applied in refs. [28, 29], shifting down all the dimensionful quantities.

[6] The connection of this dimension-two gluon condensate with the QCD vacuum has been investigated in many works [36, 37, 39, 44], as well as the appropriate definition for its renormalization scheme [38, 68].

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	MOM	$\widetilde{\text{MOM}}$	T-scheme
$g^2\langle A^2\rangle$ (GeV^2)	$5.5 \pm 0.8^{+0}_{-1.7}$	$6.0 \pm 1.3^{+0}_{-1.8}$	$5.5 \pm 1.0^{+0}_{-1.7}$
$\chi^2/d.o.f$	0.76	0.89	

Table 1. Condensate $g^2\langle A^2\rangle_{R,\mu_0}$ renormalized at $\mu_0 = 10$GeV extracted from the best fit of the lattice running couplings $\alpha_{\text{MOM}}(\mu)$ and $\alpha_{\widetilde{\text{MOM}}}(\mu)$. For comparison, the MOM T-scheme result, estimated from ref. [46] data as explained in the text, is also included. The first quoted error is purely statistical and has been computed by Jackknife and the second is a systematic uncertainty estimated as explained in section 3.

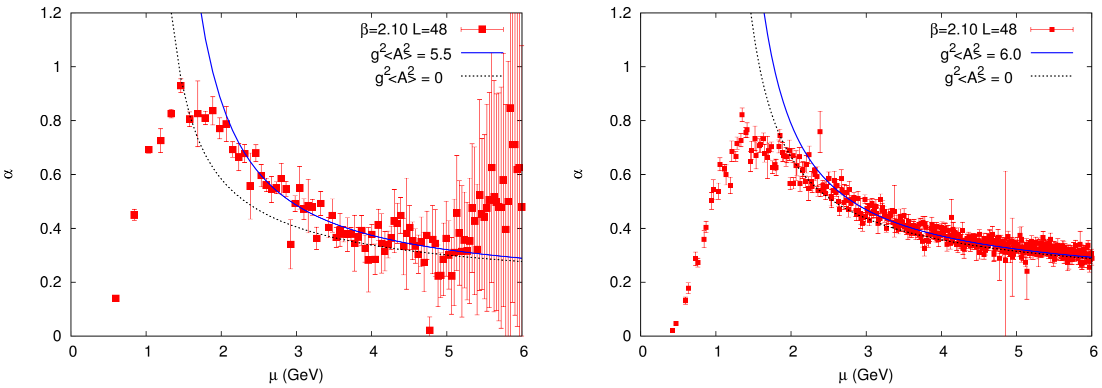

Figure 2. (Color online) Lattice data for $\alpha_{\text{MOM}}(\mu)$ [left] and $\alpha_{\widetilde{\text{MOM}}}(\mu)$ [right] vs the momenta μ in GeV. The full line shows, in both cases, the non-perturbative fit discussed in the text while the dashed line stands for the corresponding perturbative running.

scheme analysis of those papers, the beta function is expanded at the four-loop level for its integration and that the Wilson coefficient has been computed up to the $O(\alpha^4)$-order. Here, in our analysis of MOM and $\widetilde{\text{MOM}}$ couplings, we have only used a three-loop beta function and Wilson coefficient at the leading order. Indeed, the effect of including higher orders in either the perturbative part of eq. (2.1) or the Wilson coefficient is well known to reduce the value of the condensate [25, 71]. Alternatively, for the sake of a consistent comparison, we repeated the analysis of ref. [46] under the same approximation level (the best which can be here coherently attained) applied for the current one: the perturbative coupling expanded only up to three-loops and the Wilson coefficient kept at the leading-logarithm approximation. One then obtains

$$g^2\langle A^2\rangle = 5.5(1.0)\ \text{GeV}^2\,, \tag{3.1}$$

always at the renormalization point $\mu = 10\,$GeV. This last result happens to be exactly the same as the one for MOM and to lie in the same ballpark as that for $\widetilde{\text{MOM}}$, as can be seen in table 1. Indeed, one can also apply both $\Lambda_{\overline{\text{MS}}}$ from [46] and $g^2\langle A^2\rangle$ from eq. (3.1) to eq. (2.1) and, without free parameters to be fitted, account for the lattice data for MOM and $\widetilde{\text{MOM}}$ couplings with χ^2/d.o.f. that would be then 1.37 for the latter and 0.76 for the former.

It is worthwhile to emphasize that refs. [28, 29, 46] exploited the same (800) lattice configurations for the gauge fields at a bare coupling, $\beta = 2.1$, here analysed, but also configurations at $\beta = 1.90$ for three different light-quark twisted masses (500 each) and at $\beta = 1.95$ (150). The latter gives us the grounded conviction that lattice artefacts are properly under control in obtaining $\Lambda_{\overline{\rm MS}}$ and $g^2\langle A^2\rangle$ from the Taylor coupling, as done in eq. (3.1). Therefore, that eq. (2.1) successfully describes the MOM and $\widetilde{\rm MOM}$ coupling data for a given momentum window, with the same $\Lambda_{\overline{\rm MS}}$ and $g^2\langle A^2\rangle$, strongly indicates that lattice artefacts appear to be negligible also for them, after $H(4)$-extrapolation, within such a window.

3.2 The ratio of the Wilson coefficients

That the condensates obtained from both MOM and $\widetilde{\rm MOM}$ takes the same value is a demanding result, as the Wilson coefficient is three times larger for the former than for the latter. This implies that deviations from the perturbative behaviour should be very different in both cases, being consistent with the ratio of 3 given by eq. (2.6). This can be seen in figure 3(a) and, otherwise presented, as follows:

Eq. (2.1), in the leading logarithm approximation, and eq. (2.6) left us with:

$$\frac{\dfrac{\alpha_{\rm MOM}(\mu^2)}{\alpha^{\rm pert}_{\rm MOM}(\mu^2)} - 1}{\dfrac{\alpha_{\widetilde{\rm MOM}}(\mu^2)}{\alpha^{\rm pert}_{\widetilde{\rm MOM}}(\mu^2)} - 1} \;=\; \frac{c_{\rm MOM}}{c_{\widetilde{\rm MOM}}} + O\left(\alpha, \frac{1}{\mu^2}\right) \;=\; 3 + O\left(\alpha, \frac{1}{\mu^2}\right) , \tag{3.2}$$

which provides with a very demanding consistency check for the OPE and SVZ sum-rules approach, which is totally equivalent to the compatibility of condensates in table 1. To perform this check, we need to compute eq. (3.2)'s l.h.s. from the lattice data for the $\widetilde{\rm MOM}$ and MOM three-gluon coupling and their perturbative predictions obtained again by the integration of the beta function with the coefficients given in eq. (2.4), the ratios of Λ's in eq. (2.5) and $\Lambda_{\overline{\rm MS}} = 314\,{\rm MeV}$ from ref. [46]. However, as the lattice momenta for $\widetilde{\rm MOM}$ and MOM differ, and aiming at employing as large a statistics as possible, eq. (3.2)'s l.h.s. will be indirectly computed by fitting both numerator and denominator, within as large as possible a momentum domain for each, to

$$\frac{\alpha_R(\mu^2)}{\alpha_R^{\rm pert}(\mu^2)} - 1 \;=\; a_R \,\frac{\left(\alpha_R^{\rm pert}(\mu^2)\right)^{0.27}}{\mu^2} , \tag{3.3}$$

as suggested by eq. (2.1), with a_R as the only free parameter to be fitted. This can be seen in the plots of figure 3(b), where it clearly appears that, as the running given by eq. (3.3)'s r.h.s. is near the same for both schemes, $a_{\rm MOM}$ is to be rather larger than $a_{\widetilde{\rm MOM}}$. Thus, once both parameters are fitted, they can be applied to compute eq. (3.2)'s l.h.s.,

$$\frac{\dfrac{\alpha_{\rm MOM}(\mu^2)}{\alpha^{\rm pert}_{\rm MOM}(\mu^2)} - 1}{\dfrac{\alpha_{\widetilde{\rm MOM}}(\mu^2)}{\alpha^{\rm pert}_{\widetilde{\rm MOM}}(\mu^2)} - 1} \simeq \frac{a_{\rm MOM}}{a_{\widetilde{\rm MOM}}} = \frac{2.28(34)\ {\rm GeV}^2}{0.83(19)\ {\rm GeV}^2} = 2.7(8) , \tag{3.4}$$

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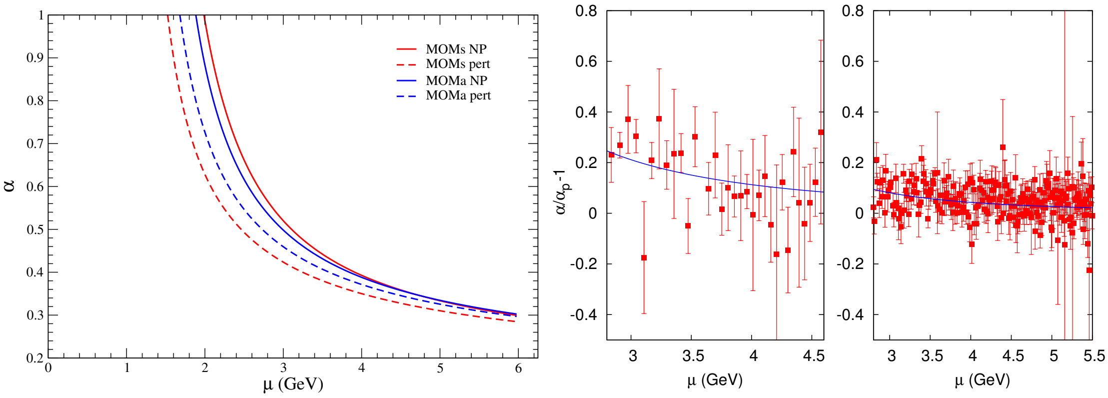

(a) (b)

Figure 3. [Color online] (a) For the sake of comparison, the perturbative (dashed) and OPE non-perturbative MOM (red) and $\widetilde{\text{MOM}}$ (blue) predictions are displayed together. (b) Ratio $\alpha/\alpha^{\text{Pert}} - 1$ for the MOM (left) and $\widetilde{\text{MOM}}$ (right) schemes vs the momenta μ in GeV. The lines show the non-perturbative fit according to eq. (3.3) discussed in the text.

that compares remarkably well with eq. (3.2)'s r.h.s. evaluated through the OPE results given by eq. (2.6). It should be noticed that, in eq. (3.4), $\alpha_{\text{MOM}}/\alpha_{\widetilde{\text{MOM}}} = 1 + O(\alpha)$ has been applied, as corresponds to our approximation level (see eq. (3.2)). We performed the fit in a momentum window $p \in (2.8, 4.5)$GeV in the MOM case and $p \in (2.8, 5.5)$GeV in $\widetilde{\text{MOM}}$. The errors for the fitted parameters, a_R, are purely statistical and have been computed by applying the Jackknife procedure, and propagated then into the final result for the ratio.

3.3 Systematic effects

We will pay now attention to the main sources of systematic effects affecting the fitted results for the condensates and discuss their impact.

A first source of possible errors comes out from the impact of discretization lattice artefacts. We have applied, as above described, the so-called $H(4)$ extrapolation procedure to cure the data from them and have also introduced a cut to remove large momenta. In particular, we take momenta below the usual[7] bound $a(2.10)p = \pi/2 \simeq 1.6$ (around 5.5 GeV, in physical units) for the fit in the $\widetilde{\text{MOM}}$ case, while the fitting window is restricted only to momenta below $a(2.10)p \simeq 1.3$ (around 4.5 GeV) for the fit in MOM. As can be seen in figure 2, errors become larger for momenta above this cut and the typical fluctuations indicating the impact of discretization artefacts appear to be the more and more visible. furthermore, we have explicitly checked that the inclusion of higher momenta does significantly change the central value for the fitted value of the condensate, mainly because of the larger errors of those data. In particular, we have estimated the condensate

[7] The restriction of $a^2p^2 < (\pi/2)^2$ guarantees for any lattice momenta, whatever the kinematic orientation might be, that $ap_\mu < \pi/2$, lying inside the first half of the *Brillouin* zone. As the usual lattice momenta emerging from lattice perturbation theory is $\hat{p}_\mu = (2/a)\sin(ap_\mu/2)$, the latter implies that the argument for the sinus of $\hat{p}_\mu$ is always smaller than $\pi/4$.

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to be 5.6(6) GeV^2 after shifting the upper bound up to 5.5 GeV in the MOM. This latter result compared with the one in table 1, 5.5(8) GeV^2, makes us sure that the systematic effect from the cut on the fitted value of the condensate is, actually, negligible.

The main source of possible systematic uncertainties is however the truncation in perturbation theory needed to get the final OPE nonperturbative prediction in eq. (2.1). We have indeed performed the analysis presented above using three-loop expressions of the beta function in MOM and $\widetilde{\text{MOM}}$, because it is the highest known order for both cases. Anyhow, for the $\widetilde{\text{MOM}}$ scheme the forth loop coefficient is also known to be

$$\beta_3^{\widetilde{\text{MOM}}} = 20718.5\,, \tag{3.5}$$

for $N_f = 4$. It can be thus used to estimate how sensitive our results might be to higher orders in the perturbative expansion of the β function, eq. (2.2). In order to check this, we repeated the fit discussed above using the four-loop beta function for the $\widetilde{\text{MOM}}$ scheme and obtained a new value of the condensate $g^2\langle A^2\rangle = 5.5(1.1)\text{GeV}^2$ with $\chi^2/d.o.f. = 0.90$, that should be compared to the one reported in table 1 for the three-loops analysis. Both are fully compatible within errors and one can therefore conclude that the systematic error associated with the truncation of the perturbative series does not seem to be relevant for our conclusions. If we took the difference between the three and four loop results as an estimate of the systematic error associated to the fitting formula used, it would be smaller than the quoted statistical error.

In the MOM scheme, the four loop beta coefficient is unknown. However, if we assume the universality of the condensate, its knowledge extracted from the four-loop fit in the $\widetilde{\text{MOM}}$ scheme can be used and fit then eq. (2.1) to the lattice data of the coupling in MOM, with the β_3^{MOM} coefficient as the only free parameter. A similar procedure was already used in [17], and allowed for a rough estimate of the three-loop coefficient β_2^{MOM}, before its perturbative computation. We thus obtain

$$\beta_3^{\text{MOM}} = 13(5)(6) \times 10^3\,, \tag{3.6}$$

where the first error quoted is purely statistical and the second accounts for the obvious correlation of the fitted value with the one of the condensate used in the fit (we moved the value of the condensate $\pm 1\sigma$ from its central value). It is important to remark that, although we do not expect this to be a very reliable determination of the four loop β^{MOM} coefficient, yet it assures that for a wide range of values of β_3^{MOM}, the value of the condensate obtained would probably be compatible with the one provided here.

On the other hand, the approximation $R(\alpha, \alpha_0) = 1$, that results from keeping only the leading logarithm in the Wilson coefficient, also implies a truncation effect. As we above indicated, the computation of $R(\alpha, \alpha_0)$ is rather cumbersome for the three-gluon vertices and no results are available in literature. We have then proceeded consistently and used the same leading-logarithm approximation for both MOM and $\widetilde{\text{MOM}}$ schemes and for the comparison of their results with those for the Taylor coupling.

$R(\alpha, \alpha_0)$ is however known for the Taylor coupling at the $\mathcal{O}(\alpha^4)$-order and the condensate have been estimated with it and the fourth-loop β coefficient to be $g^2\langle A^2\rangle =$

3.8(6) GeV^2 [46]. This, compared to eq. (3.1), allows for a rough estimate of the total impact of the unknown β function and Wilson coefficient higher orders: around a 30 % of reduction. These are the systematic errors quoted in table 1.

4 Summary and conclusions

We used lattice gauge field configurations, generated with four twisted-mass dynamical quark flavours (two light degenerate and two heavy) within the framework of ETM collaboration, to compute the running of the QCD coupling constant, α_s, defined from the symmetric and asymmetric three-gluon vertices. This leads us to two different MOM-type renormalization schemes for the coupling, where their running with momenta, roughly from 3 to 6 GeV, has been described only after supplementing the well-known perturbative prediction with a non-perturbative correction dominated by a non-vanishing dimension-two gluon condensate in Landau gauge, $\langle A^2 \rangle$.

We found the value of the condensates extracted from the fits of the non-perturbative prediction to the lattice data for both schemes to be consistent with each other, and also consistent with that resulting from the analysis in the MOM T-scheme [46]. This provides thus with additional support for the very accurate estimate of $\Lambda_{\overline{\text{MS}}}$ obtained therein. For the non-perturbative prediction in the three cases, we used a three-loop expansion of the β function and a Wilson coefficient approximated at the leading logarithm. As the fourth-loop coefficient is available in $\widetilde{\text{MOM}}$ and Taylor schemes, and the Wilson coefficient is known at the $\mathcal{O}(\alpha^4)$-order for the latter, we have estimated the systematic impact of the higher orders beyond the truncation and found it to reduce the fitted condensates by as much as roughly a 30 %. Furthermore, we have assumed the condensates to be the same and the higher-orders for the Wilson coefficients not to differ very much and have thus provided with a rough estimate for the fourth-loop coefficient of the MOM β-function.

It is worthwhile to emphasize that nothing but finding the condensates estimated for the three schemes to lie in the same ballpark, as the one defined by statistical and systematic unertainties, is a streaking result. At the leading logarithm, the Wilson coefficient for MOM and MOM Taylor have been found to be the same, but $\widetilde{\text{MOM}}$'s results to be exactly three times smaller. Therefore, obtaining the condensates from the three fits to be roughly the same implies that the non-perturbative corrections also are three times smaller for the $\widetilde{\text{MOM}}$ than for the two other schemes. Actually, as can be seen in figure 3(a), the perturbative estimate for the coupling in $\widetilde{\text{MOM}}$ scheme is larger than the one in MOM (this is a direct consequence from the way their three-loop β coefficients in eq. (2.4) and the ratios in eq. (2.5) compare to each other), contrary to their lattice non-perturbative estimates. The OPE nonperturbative corrections for both MOM and $\widetilde{\text{MOM}}$, and their relative strength, work successfully to explain the opposite pattern shown by figure 3(a) for the perturbative and nonperturbative comparison of $\widetilde{\text{MOM}}$ and MOM couplings. Indeed, we have measured the ratio between the nonperturbative corrections, fitted from the lattice data, and found it to be 2.7(8), in good agreement with our OPE prediction of a factor 3. Higher perturbative orders beyond truncation for both the β-function and the Wilson coefficient, differing for the two schemes, will certainly refine the result but will not change

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much a factor 3, as they are estimated to reduce the fitted condensates by as much as around a 30 %. This strongly supports the OPE approach to account for the nonperturbative contributions. It is worth to recall that the prediction for this ratio is only relying on the Shifman-Vainshtein-Zakharov technology [66, 67] to compute the OPE Wilson coefficients and the universality of the involved condensate.

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